

**Magnetic-field-induced transition in a wide parabolic well superimposed with a superlattice**G. M. Gusev,<sup>1</sup> Yu. A. Pusep,<sup>2</sup> A. K. Bakarov,<sup>3</sup> A. I. Toropov,<sup>3</sup> and J. C. Portal<sup>4,5,6</sup><sup>1</sup>*Instituto de Física, Universidade de São Paulo, 135960-170 São Paulo, SP, Brazil*<sup>2</sup>*Instituto de Física, São Carlos da Universidade de São Paulo, CP 66318 CEP, 05315-970 São Carlos, SP, Brazil*<sup>3</sup>*Institute of Semiconductor Physics, Novosibirsk 630090, Russia*<sup>4</sup>*LNCMI-CNRS, UPR 3228, BP 166, 38042 Grenoble Cedex 9, France*<sup>5</sup>*INSA Toulouse, 31077 Toulouse Cedex 4, France*<sup>6</sup>*Institut Universitaire de France, 75005 Paris, France*

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We study a  $\text{Al}_x\text{Ga}_{x-1}\text{As}$  parabolic quantum well (PQW) with  $\text{GaAs}/\text{Al}_x\text{Ga}_{x-1}\text{As}$  square superlattice. The magnetotransport in PQW with intentionally disordered short-period superlattice reveals a surprising transition from electrons distribution over whole parabolic well to independent-layer states with unequal density. The transition occurs in the perpendicular magnetic field at Landau filling factor  $\nu \approx 3$  and is signaled by the appearance of the strong and developing fractional quantum Hall (FQH) states and by the enhanced slope of the Hall resistance. We attribute the transition to the possible electron localization in the  $x$ - $y$  plane inside the lateral wells, and formation of the FQH states in the central well of the superlattice, driven by electron-electron interaction.

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**I. INTRODUCTION**

The multicomponent quantum Hall system, which consists of multiple quantum wells separated by the tunneling barriers, has exhibited many interesting phenomena in the strong perpendicular magnetic field due to the interlayer electronic correlations.<sup>1</sup> The previous theoretical works suggested several possible ground states in multilayer systems. The first class of the candidate states is the spontaneous coherent miniband state in superlattice (SL) quantum Hall system.<sup>2</sup> This state is an analog of interlayer coherent state at Landau filling factor  $\nu=1$  in double-well structures.<sup>3</sup> The second class of the candidate states is the solid-state phase (Wigner crystals)<sup>4</sup> with the different configurations depending on the interlayer separation. A third type of the candidate state is staggered liquid state, which consists of independent-layer states with unequal density.<sup>2</sup> Recently, it has been argued that another ground state, so-called dimer state, is favored for large number of layers and small interlayer distance. The superlattice separates into pairs of adjacent interlayer coherent states, while such coherence is absent between layers of different pairs.<sup>5</sup>

More recently novel three-dimensional (3D) fractional quantum Hall (FQH) states in multilayer systems have been theoretically predicted.<sup>6</sup> A 3D multilayer fractional quantum Hall state with average filling  $\nu=1/3$  per layer that is qualitatively distinct from a stacking of weakly coupled Laughlin states was constructed using the parton states. This new state supports gapped fermionic quasiparticles (with charge  $e/3$ ) that might propagate both within and between the layers, in contrast to the quasiparticles in a multilayer Laughlin state which are confined within each layer.

Despite the considerable theoretical efforts and predictions of the many exotic broken-symmetry states in quantum Hall superlattice, such states have not yet been observed. The experimental challenge is the fabrication of the low-disorder superlattice. However, in Refs. 7 and 8 the breakthrough idea

to use the parabolic quantum well (PQW) with a periodic modulation to produce a high quality SL has been realized. Previously the wide PQW without SL have been used to obtain the system with flat potential and constant electronic density slab.<sup>9,10</sup> In selectively doped wide PQW the electrons are spatially separated from dopant atoms and this enhances mobility and provides an opportunity to study the clean quasi-three-dimensional system. The electron in well screens the parabolic potential, and flat potential profile of  $\sim 100$ – $400$  nm width is expected. By adding the periodic potential to this system, the clean SL can be obtained. The Fig. 1 shows schematically empty and partially full parabolic well with periodical and disordered SL. Indeed in Refs. 7 and 8 it has been demonstrated that the mobility of electrons in PQW superimposed with periodical superlattice is drastically enhanced in comparison with conventional  $\text{GaAs}/\text{Al}_x\text{Ga}_{x-1}\text{As}$  superlattices.<sup>11,12</sup> However, neither the coherent states in PQW with superlattice nor other many-body effects have not yet been observed.

In the present paper we report the magnetotransport measurements in PQW with periodical and intentionally disordered short-period SL. In strong magnetic field we observed numerous well-developed plateaus in the Hall resistance at

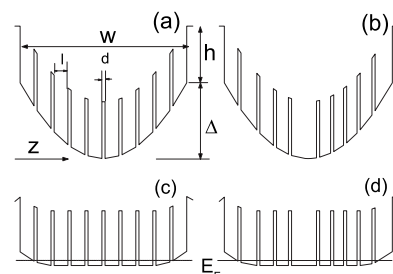


FIG. 1. The profile of the conduction-band edge of an (a) and (b) empty and (c) and (d) partially filled parabolic well with (a) and (c) periodic and (b) and (d) intentionally disordered superlattices.  $E_F$  is the Fermi level.

TABLE I. The sample parameters.  $W$  is the well width,  $n^+$  is characteristic bulk density of PQW,  $n_s$  is the electron density,  $W_{eff}$  is the effective well width, and  $\mu$  is the zero-field mobility.  $t_z$  is the interlayer coupling energy, and  $\delta$  is the interlayer disordered parameter.

Sample	$W$ (Å)	$n^+$ ( $10^{16}$ cm $^{-3}$ )	$n_s$ ( $10^{11}$ cm $^{-2}$ )	$W_{eff}$ (Å)	$\mu$ (cm $^2$ /Vs)	$t_z$ (meV)	$\delta$
A	2400	2.5	3.7	1480	136000	No superlattice	No superlattice
B	2400	2.5	3.7	1480	200000	1.5	0
C	2400	2.5	3.7	1480	186000	1.5	4.73
D	2400	2.5	3.7	1480	156000	1.5	8.5
E	2400	2.5	3.7	1480	209750	1.5	13.5

fractional filling factors  $\nu < 1$  with deep minima in the longitudinal resistance. Surprisingly, the slope of the Hall resistance is dramatically enhanced above the critical magnetic field, which corresponds to  $\nu \approx 3$ . We interpret both these effects as signaling a transition from thick slab electronic charge distribution at  $\nu > 3$  to individual well distribution at  $\nu < 3$  due to electron-electron interaction. We believe that the new state has a modulation of the charge in the  $z$  direction, perpendicular to the superlattice (staggered like state), and at the same time electrons in lateral wells are localized by impurities and does not contribute to the Hall and longitudinal conductivities. The electrons in the central well of the superlattice form FQH states which are clearly seen in experiments. The transition is absent in PQW without superlattice.

## II. EXPERIMENTAL RESULTS AND DISCUSSIONS

The samples were made from parabolic quantum well grown by molecular-beam epitaxy. It included a 2400-Å-wide parabolic  $\text{Al}_c\text{Ga}_{c-1}\text{As}$  well with  $c$  varying between 0 and 0.29, bounded by undoped  $\text{Al}_b\text{Ga}_{b-1}\text{As}$  spacer layers with  $\delta$ -Si doping on two sides.<sup>13</sup> Assuming as [001] the growth direction, and taking as  $z=0$  the position of the pure GaAs material, an effective harmonic potential is given by  $U = m^* \Omega^2 z^2 / 2$  with  $\Omega = a(2/m^*)^{1/2}$  and effective mass  $m^*$ , when a composition profile  $c(z) = az^2$  is achieved. We fabricated samples with parabolic well of width  $W = 2400$  Å and height  $h = 210$  meV. The characteristic bulk density is given by equation  $n_s = \Omega_0^2 m^* \epsilon / 4 \pi e^2$ . The effective thickness of the electronic slab can be obtained from equation  $W_{eff} = n_s / n_+$ . For partially filled quantum well  $W_{eff}$  is smaller than the geometrical width of the well  $W$ .

In addition we produced several PQWs with periodic and aperiodic square modulations on it. The total number of ten weakly coupled wells was embedded in the parabolic quantum well. They consisted of wells with thickness  $l = m$  monolayers (MLs) and barriers with fixed thickness  $d = 15$  ML. The interlayer coupling energy  $t_z = 1.5$  meV was calculated according to the effective-mass approximation in the periodic superlattice with  $l = 65$  ML. The randomization was achieved by a random variation of the layer thickness  $m$  around the nominal value  $m = 65$  ML according to a probability distribution which is obtained from a Gaussian probability density for the electron energy in the isolated well. The Gaussian is centered at the value of the electron energy cor-

responding to the 65 ML well, and it is characterized by its full width at half maximum  $\Delta$  (disorder energy). The strength of the interlayer disorder is characterized by the parameter  $\delta = \Delta / t_z$ . No coherent interlayer tunneling is expected at  $\delta > 1$ . The mobility of the electron gas in our samples was  $(140-200) \times 10^3$  cm $^2$ /Vs and density— $3.7 \times 10^{11}$  cm $^{-2}$ . The parameters of the samples are shown in Table I. Since the effective width of the wide parabolic well is smaller than the geometrical width, our superlattice structures have only 6–7 quantum wells filled by electrons. The test samples were Hall bars with the distance between the voltage probes  $L = 500$   $\mu\text{m}$  and the width of the bar  $d = 200$   $\mu\text{m}$ . Four-terminal resistance  $R_{xx}$  and Hall  $R_{xy}$  measurements were made down to 50 mK in a magnetic field up to 15 T. The sample was immersed in a mixing chamber of a top-loading dilution refrigerator. The measurements were performed with an ac current not exceeding  $10^{-7}$ – $10^{-8}$  A.

Figure 2 shows the longitudinal and Hall resistances for conventional PQW (a) and PQW with SL (b) measured at  $T = 50$  mK. We find the striking difference in the properties of these two structures. First, the conventional PQW exhibits the integer quantum Hall effect in the strong magnetic field, as is expected for quasi-two-dimensional system with moderate mobility,<sup>14</sup> while the PQW with disordered SL reveals well-developed FQH effect unexpected for such sample quality. From the comparison of the strengthening of the minima at fractional filling factors with available data for heterostructures with the same density,<sup>15</sup> we estimate the mobility  $\mu \approx 1.5-2 \times 10^6$  cm $^2$ /Vs, which is almost in one magnitude higher than zero-field mobility in our samples (see Table I). Second, the slope of the Hall resistance in superlattice structure is enhanced in two times above the critical magnetic field  $B_c$ , which is below corresponding total Landau filling factor  $\nu = 2$  (according to the low-field Hall data) or  $\nu = 1$  (considering high-field Hall resistance). Note that the electron density, and low-field Hall resistance in both samples are the same. Finally, the behavior of the Shubnikov–de Haas (SdH) oscillations at low magnetic field is also changed dramatically in SL structure in comparison with conventional PQW. We found resemblance between low-field SdH and quantum Hall features and that from low mobility SL.<sup>11</sup> For example, the Hall resistance exhibits the quantum Hall effect at integer filling factors per layer, as expected for multilayer Hall system. However, in the present paper we focus on the high-field behavior.

The data presented in Fig. 2 provide the strong evidence for the existence of the phase transition near the total filling

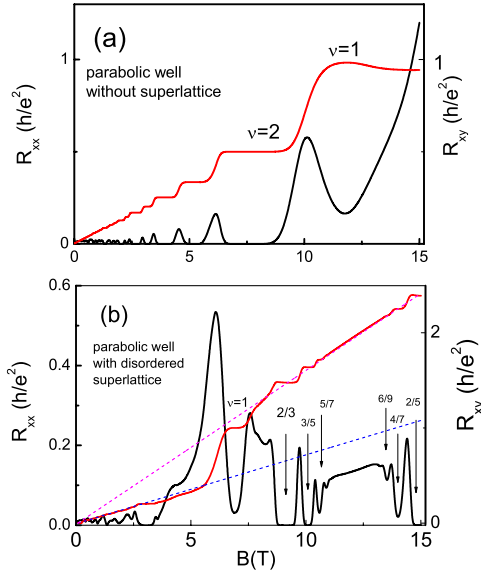


FIG. 2. (Color online) Longitudinal (black) and Hall (red) resistances as functions of the perpendicular magnetic field for a parabolic quantum well (a) without superlattice (sample A) and (b) with disordered superlattice (sample E). Filling factors determined from the Hall resistance are labeled. FQH states are marked with arrows. The dashed lines correspond to the linear extrapolation of the low-field and high-field Hall resistances.

factor  $\nu=3$ . The transition occurs in narrow interval of magnetic field  $\Delta B \sim 1.1$  T. Striking mobility enhancement, excess Hall resistance, and consequently, decrease in the total electron density cannot be explained by trivial carriers freeze-out model, which is expected to vary gradually with magnetic field. In addition, the PQW without SL [Fig. 2(a)] shows the conventional quantum Hall effect behavior; therefore the possibility that enhanced Hall coefficient in superlattice structure is described by magnetic freeze out seems very unlikely.

Figure 3 shows the temperature dependence of  $R_{xx}$  and  $R_{xy}$  in PQW superimposed with disordered SL. We may see crossover between two distinct regimes in the magnetoresistance evolution with  $T$ . At high-temperature ( $T=1$  K) magnetoresistance reveals the strong peak at  $B=8$  T, which starts to decrease with decreasing temperature, and below  $T \approx 300$  mK we observe the emergence of the FQH states. Notice that the Hall resistance demonstrates the broad transition between different slopes. In low-temperature regime  $T < 200$  mK the emergent FQH states are improved, and the plateaus in the Hall resistance, accompanied by vanishing longitudinal resistances  $R_{xx}$ , become fully developed. Such crossover may indicate that there exists competition between two different many-body states in our system with temperature evolution. At high-temperature electrons in extreme wells are only partially localized (as we can see from the Hall effect) and therefore, contribute to the conductivity. Note, however, that these high-temperature states cannot be simply described by the conductivity of the parallel channels from different independent well since it should result in the low total resistivity due to a dominant contribution from the high mobility central well. We may speculate here that this

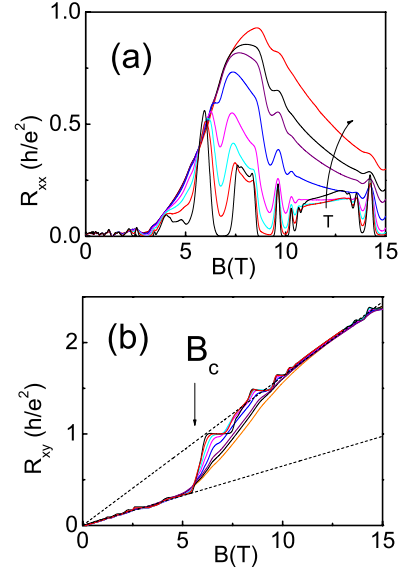


FIG. 3. (Color online) (a) Longitudinal and Hall resistances of a parabolic quantum well with disordered superlattice (sample E) for different temperatures  $T$  (from top to bottom): 950 mK (red), 750 mK (black), 700 mK (purple), 520 mK (blue), 475 mK (magenta), 320 mK (cyan), 240 mK (red), and 90 mK (black). (b) Filling factors determined from the Hall resistance are labeled. The dashed line corresponds to the linear extrapolation of the low-field Hall resistance. Arrow shows the critical magnetic field when transition occurs.

state is a many-body collective state, similar to the charge density wave state. At lower temperature this state is destroyed because of the localization of the electrons in the extreme quantum wells in SL, whereas the electrons in the central well form FQH states. Below we discuss in more details all possible ground states in multilayer quantum Hall system.

Finally, we should note that magnetotransport properties of PQW superimposed with superlattice strongly depends on the interlayer disordered parameter  $\delta$ . Figure 4 shows the of  $R_{xx}$  and  $R_{xy}$  traces for PQW with periodic and disordered superlattice. We see clear similarity in the anomalous features between the traces in Fig. 2(b) and that from another PQW with larger parameter  $\delta$  shown in Fig. 2(b). On the other hand, no similarity between magnetoresistance curves has long been seen in PQW with periodic and disordered SL. The longitudinal resistance in PQW with periodic superlattice is dramatically increases above 12 T, and we were not able to measure it. The inset shows the dependence of the slope on the Hall resistance  $R_{xy}/B$  on the parameter  $\delta$ . We see that the electron density in the strong magnetic field has a tendency to freeze out with decrease of the disordered parameter. Although not definitive, this provides some evidence that electrons in PQW with periodic SL and in strong magnetic field are localized by impurities in the  $x$ - $y$  plane and do not contribute to the Hall conductivity.

In the following we discuss the possible origin of the transition from multilayer quantum Hall system to the state with partially localized electrons. Several theoretical models conclude that the quantum Hall superlattice undergoes a

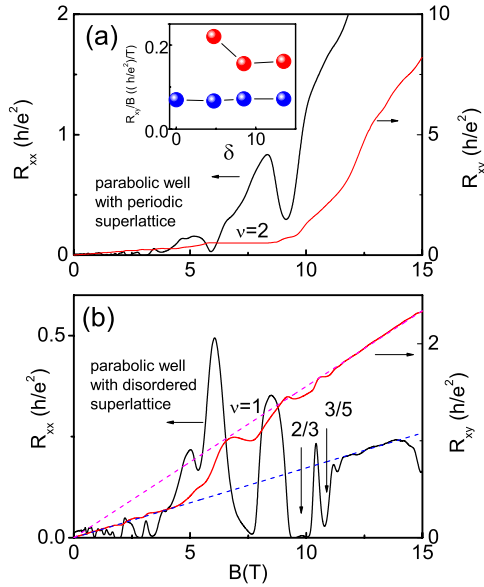


FIG. 4. (Color online) Longitudinal (black) and Hall (red) resistances as functions of the perpendicular magnetic field for a parabolic quantum well with periodic superlattice (a) (sample B) and with disordered superlattice (b) (sample D). Filling factors determined from the Hall resistance are labeled. The dashed lines corresponds to the linear extrapolation of the low-field Hall (blue) and high-field (magenta) resistances. FQH states in (b) are marked with arrows. Inset—slope of the Hall resistance at low (blue points) and high (red points) magnetic fields as a function of superlattice disorder parameter.

phase transition.<sup>2,4,5,16,17</sup> The final nature of the new many-body state depends on the parameters of the systems, such as layer separation and interlayer coupling. Enhance of the Hall resistance slope and emergence of the FQH states shown in Figs. 2–4 make it tempting to postulate that the origin of the new ground state is the electron-electron interaction which favors staggered liquid state, which consists of independent-layer states with unequal density.<sup>2</sup> From this scenario it follows that the ground state has a modulation of the charge in the  $z$  direction (staggered states), which is equivalent to the formation of the charge density wave state in three-dimensional electron system (3DES) in the strong magnetic field.<sup>18,19</sup> At the same time electrons are localized by impurities in  $x$ - $y$  plane. This state is responsible for divergence of  $R_{xx}$  and  $R_{xy}$  in strong magnetic field in PQW with periodic SL [Fig. 4(a)]. In PQW with intentionally disordered SL we break the symmetry between quantum wells, since we introduces aperiodic square modulation in  $z$  direction. For example, as we can see in Fig. 1, the central well is wider than the lateral wells. In this case it is very likely, that the electronic density in the central well is higher than in the others, therefore corresponding filling factors is sufficient to form FQH states. At the same time, local filling factors in the other wells are too small for FQH states and localized by the impurities. In addition the mobility in the central well is much higher than in the lateral wells, since the Al composition, which is mainly responsible for electronic scattering in parabolic well, is absent in the center of the well. Both these factors, density and mobility, separate the properties of the

individual quantum wells and result to the formation of FQH states in the center well, and electrons localized in  $x$ - $y$  plane, in the lateral wells. Note that the alternative possible description for states in the individual wells may be formation of the crystalline structure in  $x$ - $y$  plane pinned by impurities, which is equivalent to formation of the charged Kaplan-Glasser rods parallel to the magnetic field in 3DES case.<sup>18</sup>

Note that transition starts after total Landau filling factor 3, below filling factor 2, while staging transition is expected at arbitrary  $\nu$  per layer. For example, in a stage- $n$  state electrons would occupy every  $n$ -quantum well. Therefore  $n=2$  case corresponds to the completely full layer with  $\nu=1$ , and empty layer with  $\nu=0$ .<sup>2</sup> Assuming  $n=2$  stagelike state, we obtain 6–7 quantum well filled by electrons at low magnetic field before transition, in agreement with our previous estimations.

It is worth noting that in the strong magnetic field after transition the 50% of electron in superlattice are localized in the central well. Obviously, we do not see the contribution to  $R_{xx}$  and  $R_{xy}$  from the nearest neighbors to the central well. Note, that it would be expected that due to the small content of Al, the mobility of electrons in nearest-neighbor quantum wells is sufficient to form FQH states, which are absent in the experimental curves. It may confirm that the new ground state is a staggered state, and nearest neighbors to the central well are almost empty. In this case, it is expected that only 3–4 lateral wells are occupied by electrons. It provides the local filling factor in each individual well  $\nu^* < 1/5$  at  $B > 9$  T, which corresponds to the Anderson localization regimes.

We should make an important point when considering Anderson localization in the Hall insulator regime. The theoretical models argued that in the Hall insulator phase for non interacting<sup>20</sup> and for interacting electrons<sup>21</sup>  $\sigma_{xx}=0$ ,  $\sigma_{xy}=0$ , but  $\rho_{xy}=B/ne$ , where  $n$  is the total electron density, which is consistent with observations in Anderson localization and Wigner crystal regime.<sup>22,23</sup> Therefore, neither electron localization nor crystallization is expected to not change the Hall resistance in the quantum Hall system, which disagrees with our results. This problem deserves further theoretical study.

We also remark on the similarities between Hall resistance traces in present work and that from wide  $n$ -type ( $W > 4000$  Å) and  $p$ -type PQWs without superlattices reported in previous paper.<sup>24</sup> Authors argued that the width of the electronic and hole slabs in PQW shrinks in the strong magnetic field due to the Hartree and exchange-correlation terms, which results to the partial charge transfer from the PQW to the impurity level. This scenario is supported by previous calculations of the charge density profile in PQW in the strong magnetic field.<sup>25</sup> However, self-consistent calculations can explain only  $\sim 2\%$  decrease of the density with  $B$  in 2400-Å-wide PQW without SL, and fail to explain strong increase of the density in PQW superimposed with SL. The role of the superlattice potential in this model also remains unclear.

### III. CONCLUSION

Parabolic wells with periodic and aperiodic superlattice exhibit strong enhancement of the Hall resistance above the

critical magnetic field. In samples with disorder SL the clear evidence of the magnetic field induced transition is observed, which is characterized by two times decrease in the density and appearance of the FQH states unexpected for low sample quality. We attribute this transition to separation of the electrons into two groups: one group of electrons in the central well with highest mobility sufficient to form FQHE states and another group of low density and mobility electrons in the lateral wells, which are localized in  $x$ - $y$  plane and, therefore, do not contribute to the Hall resistivity. The transition

occurs due to electron-electron interaction which favors the redistribution of electrons in the individual wells, very likely with unequal density (staging transition).

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